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. $\sin^2 (2x + 5)$

Solution:-

2. sin 3x cos 4x

Solution:-

By standard trigonometric identity sinA cosB = ½ {sin(A + B) + cos(A - B)}

$$\int \sin 3x \cos 4x dx = \frac{1}{2}\int\left\{\sin\left(3x+4x\right)+\sin\left(3x-4x\right)\right\}dx$$

On simplifying,

$$= \frac{1}{2}\int\left\{\sin 7x+\sin\left(-x\right)\right\}dx$$

$$= \frac{1}{2}\int\left\{\sin 7x-\sin x\right\}dx$$

Splitting the integrals, we have,

$$= \frac{1}{2}\int \sin 7x dx-\frac{1}{2}\int \sin x dx$$

On integrating, we get,

$$= \frac{1}{2}\left(\frac{-\cos 7x}{7}\right)-\frac{1}{2}\left(-\cos x\right)+C$$

$$= \frac{-\cos 7x}{14}+\frac{\cos x}{2}+C$$

By standard trigonometric identity sinA cosB = ½ {sin(A + B) + cos(A - B)}

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3. cos 2x cos 4x cos 6x

Solution:-

By standard trigonometric identity cosA cosB = ½ {cos(A + B) + cos(A – B)}

$$\int \cos 2x \cos 4x \cos 6x\,dx = \int \cos 2x \left[\frac{1}{2}\left\{\cos(4x+6x)+\cos(4x-6x)\right\}\right]dx$$

$$= \frac{1}{2}\int\left\{\cos 2x \cos 10x + \cos 2x \cos(-2x)\right\}dx$$

We know that, cos(-x) = cos x,

$$= \frac{1}{2}\int\left\{\cos 2x \cos 10x + \cos^2 2x\right\}dx$$

Again by, standard trigonometric identity cosA cosB = ½ {cos(A + B) + cos(A - B)} and $\cos^2 2x$ = (1 + cos4x)/2

$$= \frac{1}{2}\int\left[\left\{\frac{1}{2}\cos(2x+10x)+\cos(2x-10x)\right\}+\left(\frac{1+\cos 4x}{2}\right)\right]dx$$

On simplifying, we get,

$$= \frac{1}{4}\int(\cos 12x + \cos 8x + 1 + \cos 4x)dx$$

By integrating,

$$= \frac{1}{4}\left[\frac{\sin 12x}{12}+\frac{\sin 8x}{8}+x+\frac{\sin 4x}{4}\right]+C$$

$$\int \cos 2x \cos 4x \cos 6x dx = \int \cos 2x\left[\frac{1}{2}\{\cos(4x+6x)+\cos(4x-6x)\}\right]dx$$

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We know that, cos(-x) = cos x,

$$= \frac{1}{2}\int\{\cos 2x \cos 10x + \cos^2 2x\}dx$$

Again by, standard trigonometric identity cosA cosB = ½ {cos(A + B) + cos(A - B)} and $\cos^2 2x = (1 + \cos 4x)/2$

$$= \frac{1}{2}\int\left[\left\{\frac{1}{2}\cos(2x+10x)+\cos(2x-10x)\right\}+\left(\frac{1+\cos 4x}{2}\right)\right]dx$$

On simplifying, we get,

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